

Problem 2

(a) Find standard deviation of the distribution
 when an example is selected at random, would show a distribution more than 1 std away from the average.
 (b) What is the probability that a photo-

Soln. (a) $f(x) = \frac{1}{2\sqrt{hx}}$, $0 \leq x \leq h$

$$\frac{\int_h^0 x^2 f(x) dx}{\int_h^0 f(x) dx} - \left(\frac{\int_h^0 x f(x) dx}{\int_h^0 f(x) dx} \right)^2$$

$$\int_h^0 x^2 f(x) dx = \int_h^0 \frac{x^2}{2\sqrt{hx}} dx = \frac{1}{2\sqrt{h}} \int_h^0 x^{3/2} dx$$

$$= \frac{1}{2\sqrt{h}} \left[\frac{2}{5/2} x^{5/2} \right]_h^0 = \frac{1}{2\sqrt{h}} \cdot \frac{1}{5} h^{5/2} = \frac{h^2}{5}$$

$$\int_h^0 x f(x) dx = \int_h^0 \frac{x}{2\sqrt{hx}} dx = \frac{1}{2\sqrt{h}} \int_h^0 x^{1/2} dx = \frac{1}{2\sqrt{h}} \cdot \frac{2}{3} h^{3/2} = \frac{h}{3}$$

$$\sigma^2 = \frac{h^2}{5} - \left(\frac{h}{3} \right)^2 = \frac{4h^2}{45}, \sigma = \frac{2\sqrt{h}}{3\sqrt{5}}$$

(b) $P = P(\langle x \rangle + \sigma) = ?$, $\langle x \rangle = \frac{h}{3}$

$$P = \int_{\frac{h}{3}}^0 \frac{1}{2\sqrt{hx}} dx = \left[\frac{1}{\sqrt{hx}} \right]_{\frac{h}{3}}^0 = \frac{1}{\sqrt{h \cdot 0}} - \frac{1}{\sqrt{h \cdot \frac{h}{3}}} = \frac{1}{\sqrt{h}} - \frac{1}{\sqrt{\frac{h^2}{3}}} = \frac{1}{\sqrt{h}} - \frac{1}{\frac{h}{\sqrt{3}}} = \frac{1}{\sqrt{h}} - \frac{\sqrt{3}}{h}$$

$$= \frac{1}{\sqrt{h}} \left(\frac{h}{3} + \sigma \right)^{-1/2}$$

$$= \frac{1}{\sqrt{h}} \left(\frac{h}{3} + \frac{2\sqrt{h}}{3\sqrt{5}} \right)^{-1/2}$$

$$= \frac{1}{\sqrt{h}} \sqrt{3} (2 + \sqrt{5})^{-1/2}$$