Date 14

(c) $\langle x \rangle = A^2 \int_{-\infty}^{\infty} dx\, x \cdot e^{-2am/\hbar\, x^2} = 0$ (odd f^n)

$$\langle x^2 \rangle = \frac{\hbar}{2 \cdot 2am} = \frac{\hbar}{4ma}.$$

$$\langle p \rangle = \frac{\hbar}{i} \int dx\, A \cdot e^{-amx^2/\hbar} e^{ait} \frac{\partial}{\partial x}\left(A e^{-amx^2/\hbar} e^{-ait}\right)$$

$$= \frac{\hbar}{i} \int dx\, e^{-amx^2/\hbar}\left(-\frac{2ma}{\hbar} x e^{-\cdots}\right)$$

$$= \frac{\hbar}{i} A^2 \int dx \left(-\frac{2ma}{\hbar}\right) x e^{-\frac{2ma}{\hbar}x^2} = 0$$

$$\langle p^2 \rangle = \int dx\, A e^{-a\left(\frac{mx^2}{\hbar} - it\right)} \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2 \psi$$

$$\frac{\partial^2}{\partial x^2}\left(e^{-\frac{am}{\hbar}x^2}\right) = \frac{\partial}{\partial x}\left(-2\frac{ma}{\hbar} x e^{-am/\hbar\, x^2}\right)$$

$$= -\frac{2am}{\hbar}\left(e^{-\frac{amx^2}{\hbar}} - 2x^2 \frac{ma}{\hbar} e^{-am/\hbar\, x^2}\right)$$

$$= -\frac{2am}{\hbar} e^{-\frac{am}{\hbar}x^2}\left(1 - 2x^2 \frac{ma}{\hbar}\right)$$

$$\langle p^2 \rangle = \cancel{\frac{2ma}{\hbar}}$$

$$= \hbar^2 A^2 \int dx\, e^{-\frac{2amx^2}{\hbar}} \cdot 2\frac{ma}{\hbar}\left(1 - 2\frac{ma}{\hbar}x^2\right)$$

$$= \hbar^2\left[\frac{2ma}{\hbar} - A^2 \int dx\, \frac{4m^2a^2}{\hbar^2} \cdot x^2 \cdot e^{-\frac{2am}{\hbar}x^2}\right]$$

$$= 2ma\hbar - ma\hbar = ma\hbar.$$