Normalization: $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$, $|\Psi(x,t)|^2 = \rho(x,t)$

<u>Proof of</u> Conservation of normalization over time.

$$\frac{d}{dt}\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = \int_{-\infty}^{\infty} \frac{\partial}{\partial t}|\Psi(x,t)|^2 dx$$

Product rule $\frac{\partial}{\partial t}|\Psi|^2 = \frac{\partial}{\partial t}(\Psi^*\Psi) = \Psi^*\frac{\partial \Psi}{\partial t} + \frac{\partial \Psi^*}{\partial t}\Psi$

<u>Schrodinger</u> eq^n: $\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar}V\Psi$

Complex Conjugate $\frac{\partial \Psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + \frac{i}{\hbar}V\Psi^*$

$$\Rightarrow \frac{\partial}{\partial t}|\Psi|^2 = \frac{i\hbar}{2m}\left(\Psi^*\frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi^*}{\partial x^2}\Psi\right) = \frac{\partial}{\partial x}\left[\frac{i\hbar}{2m}\left(\Psi^*\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right)\right]$$

or $\frac{d}{dt}\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = \frac{i\hbar}{2m}\left(\Psi^*\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right)\Big|_{-\infty}^{\infty} = 0$

as $\Psi(x,t) \to 0$ as $x \to \pm\infty$

$$\langle x\rangle = \int_{-\infty}^{\infty} x|\Psi(x,t)|^2 dx$$

Prove that $\frac{d\langle x\rangle}{dt} = -\frac{i\hbar}{m}\int \Psi^* \frac{\partial \Psi}{\partial x} dx$

$\langle v\rangle = \frac{d\langle x\rangle}{dt}$, $\langle p\rangle = m\frac{d\langle x\rangle}{dt} = -i\hbar\int\left(\Psi^*\frac{\partial \Psi}{\partial x}\right) dx$

so $\langle x\rangle = \int \Psi^* x \Psi \, dx$ and $\langle p\rangle = \int \Psi^* \frac{\hbar}{i}\frac{\partial \Psi}{\partial x} dx$

$$\langle Q(x,p)\rangle = \int \Psi^* Q\left(x, \frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi \, dx$$

$$\langle T\rangle = -\frac{\hbar^2}{2m}\int \Psi^* \frac{\partial^2 \Psi}{\partial x^2} dx$$

<u>Ehrenfest Theorem.</u>

$\langle p\rangle = m\frac{d\langle x\rangle}{dt}$, $\langle v\rangle = \frac{d\langle x\rangle}{dt}$, $\frac{d\langle p\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x}\right\rangle$